Capacity of Wireless Multi-hop Networks Using Physical Carrier Sense and Transmit Power Control

Eren Gürses, Raouf Boutaba
School of Computer Science, University of Waterloo,
Waterloo, ON N2L 3G1, Canada
{egurses, rboutaba}@uwaterloo.ca

Abstract—**In this paper, we investigate the capacity of CSMA (Carrier Sense Multiple Access) based wireless multi-hop networks with random topologies described by the hop length distribution. First we develop an analytical model for the effective link capacity as a function of transmit power adaptation policy, physical carrier sense threshold, hop length distribution and medium access probability of $p-$persistent CSMA. Secondly, we devise an *optimal transmit power control* scheme that maximizes the network capacity by adjusting the transmit power and the corresponding physical carrier sense threshold. Thereafter, it is extended for the joint optimization of these parameters with the medium access probability of CSMA. Finally we compare the optimal power control scheme with the *minimum transmit power* policy in [1]. Results show that the proposed power control scheme optimally trades off the spatial reuse (number of interfering links) to the link SIR (Signal-to-Interference Ratio) and achieves an amount of $\sim$%15 increase in network capacity when both schemes employ joint optimization.**

Index Terms—**capacity, transmit power control, physical carrier sense, wireless multi-hop networks, CSMA.**

I. Introduction

In the last decade, wireless multi-hop networks have drawn tremendous amount of attention due to their numerous potential application areas and desirable features. However, these features such as having no infrastructure requirement and being decentralized and ad hoc in nature, necessitate these networks to self-organize and self-optimize themselves in order to maximize their capacity. In their seminal work, Gupta and Kumar [2] gave a formula for the achievable transport capacity of a multi-hop ad hoc network. The analysis is done for the point-to-point coding model, which means a receiver decodes messages from only one sender and considers other simultaneous transmissions as noise. Recent works [6], [7] have shown that when nodes employ mechanisms of cooperative communication such as network coding or relaying with interference cancellation, it is possible to achieve higher bounds on transport capacity. However for the high-attenuation case where path loss exponent $\alpha > 3$, employing point-to-point coding is the order-optimal strategy [6]. Hence in this paper we consider shadowed urban areas with high attenuation factors (for ex. in simulations $\alpha = 4$) where point-to-point coding is preferable and the capacity bounds given in [2] hold. Although the bounds are binding, no specific mechanism for optimal scheduling and power adaptation is proposed in order to achieve these bounds. Our main goal in this paper is to investigate these mechanisms through physical carrier sense and transmit power adaptation in order to find the best balance between link capacities (or SIR) and the spatial reuse. In [8], it has been shown that 802.11 spatial reuse mechanisms are not effective. A recent study in [9] investigates the mechanisms to balance the spatial reuse and SINR in slotted Aloha networks. In this work, it is observed that the optimal SINR threshold for packet detection is much lower than that used in many systems. Then, in order to exploit this difference, the system is let to work closer to the SINR thresholds by allowing longer transmission distances. However, in [9] some controversial results such as encouraging the use of longer links, that conflict with the theory in [2] are reported. In [10], [11], there has been some attempts to optimize the physical carrier sense threshold. On the other hand, in [12] the relation between transmit power and carrier sense threshold has been investigated and it has been shown that spatial reuse depends on the ratio of these two entities. However none of these works has considered the joint tuning of transmit power and carrier sense. In [1], it is shown that how carrier sense threshold is adjusted according to the selected transmit power level. However, only considering the negative impact of high transmission power on spatial reuse, it is concluded that *minimum transmit power* policy is the best choice. However, besides its negative impact on spatial reuse, higher transmit power yields higher SIR and capacity when adaptive coding and modulation is employed. Considering these factors, a better trade-off between spatial reuse and transmit power maximizing the system capacity should be seeked.

In this paper, we devise an optimal transmit power and physical carrier sense threshold adaptation (i.e. control) scheme to make the best trade-off between the spatial reuse and link SIR (or capacity), that maximizes the overall network capacity. In Section II, we devise the analytical model for link capacity as a function of power adaptation function, hop length probability distribution and link access probability of the $p-$persistent CSMA. In Section III, we give the optimization framework and a possible set of functions for transmit power adaptation. Finally, in Section IV we give the simulation results and comparisons with [1] then we conclude in Section V.

II. Effective Link Capacity: Physical Carrier Sense, Transmit Power, Link Access Probability

In a wireless multi-hop network with $\mathcal{L}$ links, we represent the hop length as a random variable $\tilde{d}$ with a density function

978-1-4244-4148-8/09/$25.00 ©2009
This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE "GLOBECOM" 2009 proceedings.

$f_{\tilde{d}}(d)$. Our goal is to model the link capacities as a function of the power adaptation (i.e. control) policy in the network and the corresponding physical carrier sense threshold CS_{th} selection process which together have an impact on the spatial reuse (i.e. number of contending links) and the wireless link qualities by effecting the SIR (Signal-to-Interference-Ratio) levels.

Given the receiver sensitivity threshold RX_{th}, a transmitter s transmitting on link (s,r) of length d_{sr} should satisfy a lower bound $P_{\min}(d_{sr})$ on transmit power, for the receiver r to detect it. Then using the *path loss propagation model*, following equation holds where g and α are given as the antenna gain and path loss exponent respectively.

$$\frac{gP_{\min}(d_{sr})}{d_{sr}^{\alpha}} = RX_{th} \tag{1}$$

Assuming a power adaptation policy $\lambda(d)$ as a function of hop length d, a node s may transmit on link (s,r) at a higher power level given in (2) where $\lambda(d_{sr}){\geq}1$. In the rest of the paper for the sake of brevity, we denote the transmit power after adaptation as $P_{sr} = P(d_{sr})$ which is a function of d_{sr}.

$$P_{sr}{=}P_{\min}(d_{sr}).\lambda(d_{sr}) \tag{2}$$

After solving (1) for $P_{\min}$ and replacing in (2) we obtain the following equality.

$$\frac{gP_{sr}}{d_{sr}^{\alpha}} = \lambda(d_{sr})RX_{th} \tag{3}$$

A. Interference Range and Carrier Sense Threshold

We first define the interference set of link (s,r) as $IN_{sr} = \{(s',r') \in \mathcal{L} : (gP_{s'r'}/d_{s'r'}^{\alpha})/(gP_{sr}/d_{sr'}^{\alpha}) < \hat{\beta}\}$. The parameter $\hat{\beta} = \beta(\frac{\alpha}{\alpha-2})^{\alpha/2}$ is devised to characterize the *capture effect* of wireless modems after considering the cumulative effect of other transmitters [1]. Note that β is the capture effect model parameter for the two transmitter case [3] and simply says that if a receiver hears two conflicting transmissions (i.e. overlapping in time) where one is received β times stronger than the other, then the stronger one can be decoded by rejecting the other as noise. Hence we define IN_{sr} above as the set of links that the capture effect doesn't hold. Then we introduce the interference range $d_{IN,sr}$ of (s,r) in (4) as the maximum distance $d_{sr'}$ from transmitter s to the receiver r' of some link $(s',r') \in IN_{sr}$ (i.e. transmission on (s,r) interferes with (s',r')).

$$d_{IN,sr} = \max_{(s',r')}\{d_{sr'}\} \;\text{ s.t. } (s',r') \in IN_{sr} \tag{4}$$

For any $(s',r') \in IN_{sr}$, from the definition of IN_{sr} we replace (3) in $(gP_{s'r'}/d_{s'r'}^{\alpha})/(gP_{sr}/d_{sr'}^{\alpha}) < \hat{\beta}$ and obtain $\lambda(d_{s'r'})RX_{th}/(gP_{sr}/d_{sr'}^{\alpha}) \leq \hat{\beta}$. By arranging the terms, we obtain $d_{sr'}^{\alpha} \leq \hat{\beta}gP_{sr}/(\lambda(d_{s'r'})RX_{th})$ for all $(s',r') \in IN_{sr}$. Replacing $d_{sr'}$ in (4) and solving the maximization problem for all (s',r'), we find the interference range in (5) where

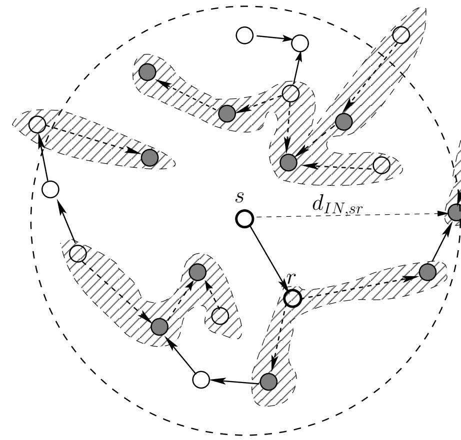

Fig. 1. Shaded regions give the Interference set IN_{sr} and the corresponding interfering distance given as $d_{IN,sr}$

$\lambda(d_{\max})$ is given as the minimum transmit power increase in a given power adaptation policy $\lambda(d)$.

$$d_{IN,sr} = \Big(\frac{\hat{\beta}gP_{sr}}{\lambda(d_{\max})RX_{th}}\Big)^{1/\alpha} \tag{5}$$

The following theorem is given directly from the definition of $d_{IN,sr}$ in (4) and IN_{sr} is illustrated in Fig.1. Note that in the figure shaded regions refer to IN_{sr} for link (s,r). And solid nodes refer to the receivers (a receiver of some interfering link), where one of it's incoming links interfere with the transmission on (s,r). Dashed links cannot transmit at the same time with (s,r) and are included in IN_{sr}.

Theorem 1 *Let $d_{IN,sr}$ be the interference range of link (s,r), then transmission on (s,r) does not interfere with the transmission on link (s',r') outside its interference range $d_{sr'}{>}d_{IN,sr}$.*

The theorem simply states that the circular region described by interference distance $d_{IN,sr}$ includes the interference set IN_{sr}, the shaded region in Fig.1. However in the circular region defined by radius $d_{IN,sr}$ there are still some nodes that (s,r) does not interfere with.

To calculate the carrier sense threshold $CS_{th,s}$ of s we use the formulation in [1]. Hence the rest of this subsection is given for the sake of completeness of the notation. The $CS_{th,s}$ of s should satisfy the condition such as (s,r) is backed off in case of a transmission on any link $(s',r') \in IN_{sr}$. In other words, the link (s,r) in Fig. 1 should not transmit when any link $(s',r') \in IN_{sr}$ transmits. We first define $P_s(s',r')$ as the received power at s due to transmission on (s',r') which is given by $P_s(s',r') = gP_{s'r'}/d_{s's}^{\alpha}$. Then by using triangle inequality $d_{s's} \leq d_{s'r'} + d_{IN,sr}$, we rewrite $P_s(s',r')$ as in (6). Finally the equality (3) implies $\frac{gP_{sr}}{d_{sr}^{\alpha}} \geq \lambda(d_{\max})\mathrm{RX}_{th}$ and we rewrite the condition on $P_s(s',r')$ as in (7).

978-1-4244-4148-8/09/$25.00 ©2009
This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE "GLOBECOM" 2009 proceedings.

$$P_s(s',r') \geq \frac{gP_{s'r'}}{(d_{s'r'}+d_{IN,sr})^\alpha} = \frac{gP_{s'r'}/d^\alpha_{s'r'}}{(1+\frac{d_{IN,sr}}{d_{s'r'}})^\alpha} \quad (6)$$

$$\geq \frac{RX_{th}\lambda(d_{\max})}{(1+\frac{d_{IN,sr}}{d_{s'r'}})^\alpha} \quad (7)$$

The carrier sense treshold of s is given as the minimum value of $P_s(s',r')$ in (7). Then the dependency of $CS_{th,s}$ to some random node s' is eliminated by replacing $d_{s'r'}{=}E\{\tilde{d}\}$.

$$CS_{th,s} = \frac{RX_{th}}{\left(1+\frac{d_{IN,sr}}{E\{\tilde{d}\}}\right)^\alpha} \quad (8)$$

The expression for $CS_{th,s}$ is exactly the same as in [1]. Different from that, we allow a transmit power P_{sr} higher than the minimum required power $P_{\min}(d_{sr})$ which results in a different expression $d_{IN,sr}$ in (5) with arbitrarily shaped (i.e. non-circular) and not connected interference set IN_{sr} as illustrated in Fig.1.

B. Links Contending for Medium Access

As the next step, we calculate the number of contending links $N_\lambda(d_{sr})$ with the transmission on outgoing link (s,r) of node s. We consider a ring with radii $(y, y+\Delta y)$. Assuming a random node s' on this ring with a randomly selected link (s',r'), then the hop length of (s',r') is a random variable and denoted as $\tilde{d}$. Then the corresponding transmit power on (s',r') is also a random variable $\widetilde{P}$ and satisfies the condition $g\widetilde{P}/y^\alpha > CS_{th,s}$ that (s,r) is contending with (s',r') to capture the medium. Then the number of links contending with (s,r) to capture the medium, is given in (9). After putting $d_{IN,sr}$ in $CS_{th,s}$ using (8) we arrange the terms and replace $g\widetilde{P}/RX_{th} = \tilde{d}^\alpha\lambda(\tilde{d})$ using (3) in order to obtain (10). Finally, using (3), we replace $gP_{sr}/RX_{th} = d^\alpha_{sr}\lambda(d_{sr})$ in (10) and obtain the resulting expression in (11).

$$N_\lambda(d_{sr}) {=} \int_0^\infty 2\pi\delta y \Pr(g\widetilde{P}/y^\alpha > CS_{th,s})dy \quad (9)$$

$$= \int_0^\infty 2\pi\delta y \Pr\left(\widetilde{d}^\alpha\lambda(\widetilde{d}) {>} \Big(\frac{y}{1{+}\frac{1}{E\{\tilde{d}\}}\big(\frac{\hat{\beta}gP_{sr}}{RX_{th}\lambda(d_{\max})}\big)^{\frac{1}{\alpha}}}\Big)^\alpha\right)dy \quad (10)$$

$$= \int_0^\infty 2\pi\delta y \Pr\left(\widetilde{d}^\alpha\lambda(\widetilde{d}) {>} \Big(\frac{y}{1{+}\frac{d_{sr}}{E\{\tilde{d}\}}\big(\frac{\hat{\beta}\lambda(d_{sr})}{\lambda(d_{\max})}\big)^{\frac{1}{\alpha}}}\Big)^\alpha\right)dy \quad (11)$$

C. Signal to Interference Ratio

Theorem 2 *Let P_{sr} be the transmit power adjusted using the policy $\lambda(d)$ and d_{sr} be the hop length of link (s,r), then the interference on link (s,r) is given by*

$$SIR_{sr} > \frac{\hat{\beta}gP_{sr}}{\lambda(d_{\max})RX_{th}E\{\tilde{d}\}^\alpha} = \frac{\hat{\beta}d^\alpha_{sr}\lambda(d_{sr})}{\lambda(d_{\max})E\{\tilde{d}\}^\alpha} \quad (12)$$

Proof: Given the link (s,r) in Fig.3, we first consider the case prior to the power adaptation. Hence, nodes transmit

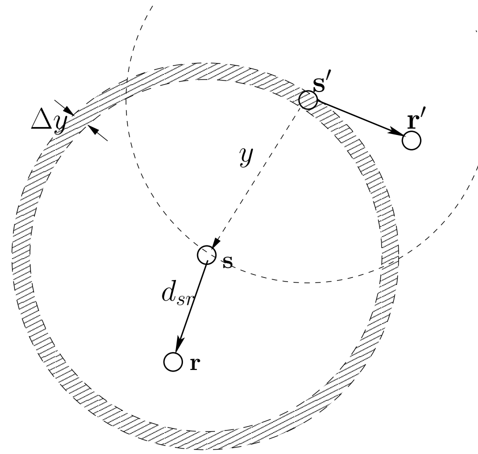

Fig. 2. Node s' of link (s',r') contending with (s,r) to access the medium

using minimum required power levels on all links (s,r) and (s',r') in Fig. 3 which are respectively denoted as $P_{\min}(d_{sr})$ and $P_{\min}(d_{s'r'})$, and satisfy the equality given in (1). Then for the transmission on (s,r), the link (s',r') is at such a distance from s that it is not silenced by s (i.e. $gP_{\min}(d_{sr})/d^\alpha_{s's} < CS_{th,s'}$) and causing interference on link (s,r).

Secondly, we consider the case after the power adaptation, where nodes s and s'' respectively transmiting on links (s,r) and (s'',r'') follow a power adaptation policy $\lambda(d)$ that increases the transmit power of a node as a function of hop length d. Hence the transmit power after adaptation is given as $P_{sr} = P_{\min}(d_{sr})\lambda(d_{sr})$ and the equality $gP_{sr}/d^\alpha_{sr} = \lambda(d_{sr}).RX_{th}$ in (3) holds. Similar to the first case we say there is such link (s'',r'') of length $d_{s''r''} = d_{s'r'}$ with transmit power $P_{s''r''}$ given in (13) which is at some further position (see Fig. 3) and is not silenced by s since $gP_{sr}/d^\alpha_{s''s} < CS_{th,s''}$.

$$P_{s''r''} = P_{\min}(d_{s'r'}).\lambda(d_{s''r''}) \quad (13)$$

Considering the power adaptation, we write the Signal-to-Interference (SIR) ratio of link (s,r) due to interfering link (s'',r'') as $\text{SIR}_{sr} = \frac{gP_{sr}/d^\alpha_{sr}}{gP_{s''r''}/d^\alpha_{s''r}}$ which is further written as below using (13).

$$\text{SIR}_{sr} = \frac{gP_{sr}/d^\alpha_{sr}}{gP_{\min}(d_{s'r'})/d^\alpha_{s'r}} \frac{1}{\lambda(d_{s''r''})} \Big(\frac{d_{s''r}}{d_{s'r}}\Big)^\alpha \quad (14)$$

In order to calculate the ratio $\frac{d_{s''r}}{d_{s'r}}$ in (14), we say $d_{s''r} = d_{IN,s''r''}$ since (s'',r'') is not silenced by s. Then, the ratio is found as follows, using (1), (4) and (13).

$$\Big(\frac{d_{s''r}}{d_{s'r}}\Big)^\alpha = \frac{\hat{\beta}gP_{s''r''}/(\lambda(d_{\max})RX_{th})}{\hat{\beta}gP_{\min}(d_{s'r'})/RX_{th}} = \frac{\lambda(d_{s''r''})}{\lambda(d_{\max})} \quad (15)$$

Then, in the rest of the proof we calculate the $\frac{gP_{sr}/d^\alpha_{sr}}{gP_{\min}(d_{s'r'})/d^\alpha_{s'r}}$. In order to obtain this ratio, we write the

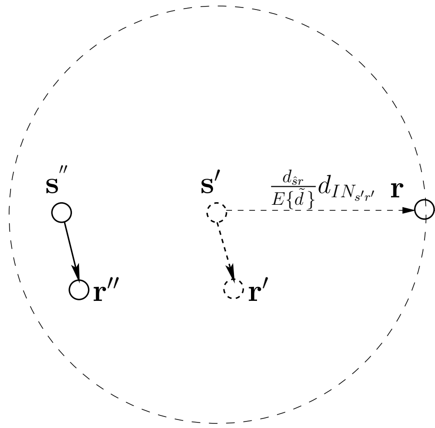

Fig. 3. Calculating SIR_{sr}: After power adaptation policy $\lambda(d)$ transmission on (s'', r'') interferes with (s, r)

interference range of s' in (16) and by placing $RX_{th} = gP_{\min}(d_{sr})/d_{sr}^{\alpha}$ in (8) we rewrite $CS_{th,s'}$ as (17).

$$d_{IN,s'r'}^{\alpha} = \frac{\hat{\beta} g P_{\min}(d_{s'r'})}{RX_{th}} \tag{16}$$

$$CS_{th,s'} = \frac{gP_{\min}(d_{sr})}{\left(d_{sr} + \frac{d_{sr}}{E\{\tilde{d}\}} d_{IN,s'r'}\right)^{\alpha}} \tag{17}$$

Writing the interference condition $gP_{\min}(d_{sr})/d_{s's}^{\alpha} < CS_{th,s'}$ together with (17) we obtain $d_{s's} > d_{sr} + \frac{d_{sr}}{E\{\tilde{d}\}} d_{IN,s'r'}$. Using this result and triangle inequality $d_{s'r} \geq d_{s's} - d_{sr}$ in Fig.3, we write;

$$d_{s'r} > \frac{d_{sr}}{E\{\tilde{d}\}} d_{IN,s'r'} \tag{18}$$

Hence by using (16) and (18) we obtain the interference at r due to transmission on s' as below:

$$\frac{gP_{\min}(d_{s'r'})}{d_{s'r}^{\alpha}} < \frac{RX_{th}}{\hat{\beta}} \Big(\frac{E\{\tilde{d}\}}{d_{sr}}\Big)^{\alpha} \tag{19}$$

Finally by replacing (15) and (19) in (14) we obtain the expression $\text{SIR}_{sr} > \frac{\hat{\beta} g P_{sr}}{\lambda(d_{\max}) RX_{th} E\{\tilde{d}\}^{\alpha}}$ which completes the proof. Equivalently we can write the expression by replacing $gP_{sr}/RX_{th} = \lambda(d_{sr}) d_{sr}^{\alpha}$ using (3). ∎

D. *Effective MAC Layer Capacity*

To construct the effective MAC layer capacity, we consider $p-$persistent CSMA as the MAC layer where the backoff time is taken from a geometric distribution with parameter p. Nodes with a packet to send will check the availability of medium by using the carrier sense mechanism. If it is sensed idle, then nodes attempt to transmit in the next slot with probability p. Under saturation conditions, the DCF (Distributed Coordinated Function) of 802.11 can be modeled as a slotted $p-$persistent CSMA [5]. In the rest of the paper we denote $p = A$ as the medium access probability on any link (s, r) employing $p-$persistent CSMA. Given A and $N_{\lambda}(d_{sr})$, we write $A\,(1 - A)^{N_{\lambda}(d_{sr})}$ as the successful medium access probability of link (s, r) where $(1 - A)^{N_{\lambda}(d_{sr})}$ is the joint probability of links contending with (s, r) not accessing the medium. Hence, the MAC layer capacity $C_{\lambda}^{MAC}(d_{sr}) = C_{sr}\, A\,(1 - A)^{N_{\lambda}(d_{sr})}$ is decided by both the MAC layer operation driven by A and entities such as physical layer link capacity C_{sr} and number of contending links $N_{\lambda}(d_{sr})$. Finally replacing SIR_{sr} (12) in $C_{sr} = W \log(K.\text{SIR}_{sr})$, we write the effective MAC layer link capacity as a function of λ, d_{sr} and probability density of $\tilde{d}$ as below where $N_{\lambda}(d_{sr})$ could be replaced with (11).

$$C_{\lambda}^{MAC}(d_{sr}) = W \log\Big(\frac{K\hat{\beta} d_{sr}^{\alpha} \lambda(d_{sr})}{\lambda(d_{\max}) E\{\tilde{d}\}^{\alpha}}\Big) A(1 - A)^{N_{\lambda}(d_{sr})} \tag{20}$$

III. Maximizing The Network Capacity

Given the probability density $f_{\tilde{d}}(d)$ of hop length, the network capacity maximization problem in (21) is defined as finding the best power adaptation policy $\lambda^*(.)$ that maximizes the mean of the MAC layer link capacities $C_{\lambda}^{MAC}(d_{sr})$.

$$\lambda^*(.) = \arg\max_{\lambda(.)} \int_0^{\infty} f_{\tilde{d}}(x) C_{\lambda}^{MAC}(x) dx \tag{21}$$

A. *Transmit Power Adaptation Policy $\lambda(d)$*

We select the power adaptation function $\lambda(d)$ from a set of functions where it is inversely proportional to the d^{α} with a scaling factor K_1/a^{α} plus a constant additive term $K_2(a, b)$. Plots of a set of possible functions for $\lambda(d)$ are given in Fig. 4 where it is ensured that $\lambda(d)$ is smaller than the maximum power increase $\lambda_{\max}(d)$ with proper selection of K_1 and K_2.

$$\lambda(d) \in \Big\{ \frac{K_1}{(ad)^{\alpha}} + K_2(a, b) \,|\, a \geq 1 \text{ and } 0 \leq b \leq 1 \Big\} \tag{22}$$

Given the maximum transmit power $P_{\max}$ and the minimum required power $P_{\min}(d)$, the maximum power increase is given as $\lambda_{\max}(d) = P_{\max}/P_{\min}(d)$. After defining $\lambda_{\text{MAX}} = \lambda_{\max}(d_{\max})$ and replacing $d_{\max}$, $P_{\max}$, λ_{MAX} in (3) we obtain $\lambda_{\text{MAX}} = \frac{gP_{\max}}{d_{\max}^{\alpha} RX_{th}}$. In order the given form of functions in (22) to span the set $\lambda(d) \leq \lambda_{\max}(d)$, they satisfy $1 \leq \lambda(d_{\max}) \leq \lambda_{\text{MAX}}$, $a \geq 1$ and $0 \leq b \leq 1$. Hence by applying these conditions, parameters K_1 and $K_2(a, b)$ are found as below.

$$K_1 = \frac{gP_{\max}}{RX_{th}} \;\; ; \; K_2(a, b) = 1 - \frac{\lambda_{\text{MAX}}}{a^{\alpha}} + b(\lambda_{\text{MAX}} - 1) \tag{23}$$

Finally by using the given family of power adaptation function in (22), we rewrite $N_{\lambda}(d_{sr})$ as follows using (11). The probability distribution function $\Pr(\tilde{d}^{\alpha} > Y)$ in the integral is obtained by using *functions of random variables* [4] from the density function $f_{\tilde{d}}(d)$ of hop length $\tilde{d}$.

$$N_{\lambda}(d_{sr}) = 2\pi\delta \times \int_0^{\infty} y \Pr\Bigg(\tilde{d}^{\alpha} > \frac{1}{K_2(a, b)} \Bigg(\Bigg(\frac{y}{1 + \frac{d_{sr}}{E\{\tilde{d}\}} \big(\frac{\hat{\beta}\lambda(d_{sr})}{\lambda(d_{\max})}\big)^{\frac{1}{\alpha}}}\Bigg)^{\alpha} - \frac{K_1}{a^{\alpha}}\Bigg)\Bigg) dy$$

978-1-4244-4148-8/09/$25.00 ©2009
This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE "GLOBECOM" 2009 proceedings.

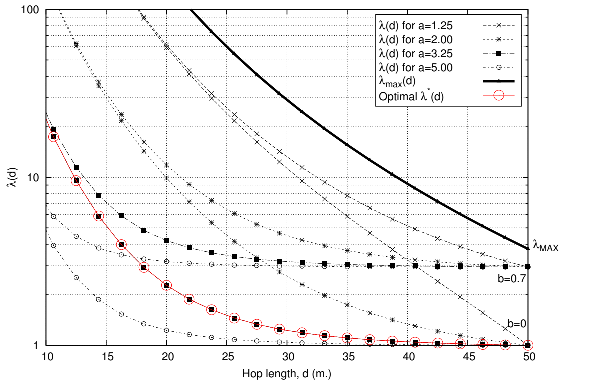

Fig. 4. For $b \in (0, 0.7)$, $a \in (1.25, 2.0, 3.25)$ power adaptation function $\lambda(d)$

TABLE I
DEFAULT SET OF PARAMETERS USED IN THE SIMULATIONS

Parameter	Value
α	4.0
β	10
K	3
δ	100 nodes/km^2
W	5 MHz
$d_{\min}, d_{\max}$	5 and 50 m
$E\{\tilde{d}\}$	27.5 m (uniform dist.)
$P_{\max}$	20 dBm (100 mW)
RX_{th}	-92 dBm (6.31×10^{-10} mW)
G_t, G_r	1 dBm
g	-38.3027 dB
A	0.10

IV. Simulation Results

In the simulations we use the set of default parameters given in Tab. I unless declared otherwise. Using MATLAB, we validated the proposed analytical model and solved the maximization problem in Sec. III with a precision of 0.05 by using binary search method over the $\lambda(.)$ function space defined by parameters (a, b). Note that the solution is obtained in a centralized fashion, given the hop count distribution.

Using the *path loss* propagation model for transmission on link (s, r) with transmit power P_{sr}, we can write the received power at distance d_{sr} as $\frac{P_{sr}G_tG_r(C/f_c)^2}{(4\pi d_0)^2 L}(\frac{d_0}{d_{sr}})^{\alpha}$. The first fractional term is the received power at distance $d_0 = 1$ m. in free space model where speed of light C=300×10^{-6} m/s and parameters such as carrier frequeny f_c=2.472 GHz, transmitter/receiver antenna gain G_t=G_r=1 dBm and system loss L=1 are given. Then after replacing d_0=1 and L=1, it is found that g=$\frac{G_tG_r(C/f_c)^2}{(4\pi)^2}$. Putting this in the received power expression in path loss model, we obtain $\frac{gP_{sr}}{d_{sr}^{\alpha}}$ as in (3). By placing all the parameters in the expression for g, it's numerical value is calculated and given in Tab. I

In the first experiment, we show and discuss the optimal power adaptation policy $\lambda^*(d)$ where the parameters δ, A and others are set to the default values. In order to obtain the optimal power adaptation function we simply conduct binary search over $0 \le b \le 1$ and $1 \le a \le a_{\max}$ by setting $a_{\max} =$

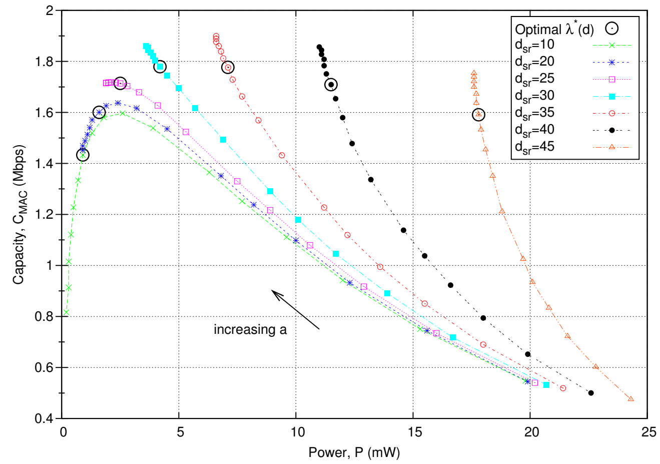

Fig. 5. For $d_{sr} \in \{10, 20, ...45\}$, capacity of links under power adaptation policies given by $a \in \{1.50, 1.60, 1.70...5.25\}$ and optimal policy $\lambda^*(d)$

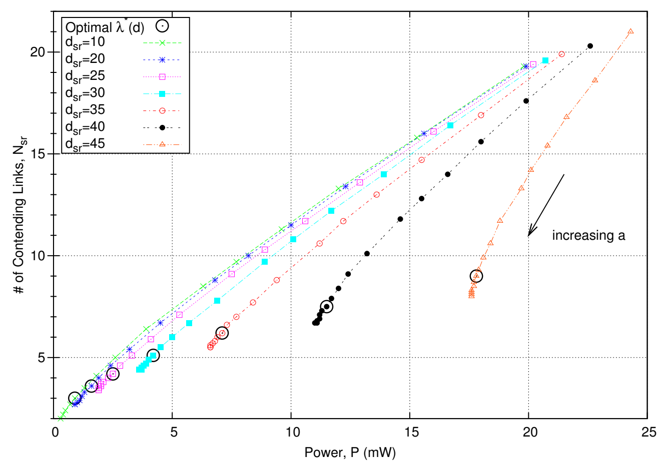

Fig. 6. For $d_{sr} \in \{10, 20, ...45\}$, num. of contending links under power adaptation policies given by $a \in \{1.50, 1.60, 1.70...5.25\}$ and the optimal policy $\lambda^*(d)$

10, and achieve optimal values as $(a^*, b^*) = (3.25, 0.0)$ where $\lambda^*(d)$ is given by (22)-(23) and plotted in Fig. 4. Note that for $b^* = 0$ the power adaptation $\lambda(d_{\max}) = 1$ using (22)-(23) which translates transmit power for the longest hop to be exactly the minimum required power $P_{\min}(d_{\max})$ regardless of the selection of a. Then the optimal power adaptation $\lambda^*(d)$ increases inversely proportional to the hop length (22)-(23).

In the next experiment, for a set of hop lengths $d_{sr} \in \{10, 20, 25, 30, 35, 40, 45\}$, we indicate the operating point of the optimal power adaptation policy $\lambda^*(dsr)$ on the plots of MAC layer link capacity C_{MAC} in Fig. 5 and number of contending links N_{sr} in Fig. 6. To maximize the *average link capacity* of the network, the optimal operating point (shown in black circles) is set to values larger than minimum power levels $P_{\min}(d_{sr})$ in Fig. 5 which leads to a higher number of contending links than the minimum power case in Fig. 6. We plot these figures for every d_{sr} for a set of power adaptation functions given by $a \in \{1.50, 1.60, 1.70, 1.80, 1.90, 2.0, 2.25, 2.50, ..., 5.0, 5.25\}$ and $b = 0$. The optimal ones are $(a^*, b^*) = (3.25, 0.0)$ as found above and indicated by large black circles in figures. In Fig. 5, we observe that for increasing values of $a = 1.50, 1.60...$ (i.e. reducing the tx. power), the capacity of longer links ($d \geq 30$) persistently increases, whereas for links $d < 25$ m. capacity increases until some value and then decreases. In Fig 5, we indicate the points achieved by the optimal policy λ^* which maximizes the mean network capacity in (21) and is given by $(a^*, b^*) = (3.25, 0.0)$ In Fig. 6 we illustrate the effect of the optimal power adaptation policy on the relation between the power and the number of contending nodes $N_\lambda(d_{sr})$. Using the definition in (11), we observe that decreasing a (i.e. increasing power) reduces the $CS_{th,s}$ and increases the probability of nodes with higher transmit power, which increases the $\Pr(g\widetilde{P}/y^\alpha > CS_{th,s})$ and the $N_\lambda(d_{sr})$ in (11). Consequently, as expected, the number of contending links increases with the tx. power. However it is important to observe that, to maximize the *average link capacity* of the random network, the system optimal is attained at transmit power levels higher then the minimum power (Fig. 5) which results in slightly more than the minimum number of contending nodes.

In the final experiment, we illustrate the two major issues regarding the analytical model in (20) and the related optimization problem (21). First, we show the impact of joint optimization of PHY and MAC layers on the network capacity. In Fig. 7 for the optimal power control case, jointly optimizing the medium access probability A with λ yields a higher capacity. By solving the problem in (21) wrt. both $\lambda(.)$ and A, it is possible to attain higher networks capacity at the jointly optimal point $A^* = 0.175$ and $\lambda^*(d)$ given by $(a^*, b^*) = (3.75, 0)$. Thereafter, with the increasing link density $\delta = 100, 200, 400$, the jointly optimal solution to (21) wrt. $\lambda(.)$ and A, reduces the link access probability A^* while keeping the power adaptation policy the same $(a^*, b^*) = (3.75, 0)$. The jointly optimal values for medium access is respectively found as $A^* = 0.175, 0.1, 0.05$ for the values $\delta = 100, 200, 400$.

Secondly, we show the impact of *optimal power control* on network capacity over the *min. power policy* which selects the minimum required power for any transmission [1]. In Fig. 7. We compare the minimum power policy to the proposed *optimal power control* scheme and for different network densities of $\delta = 100, 200, 400$, the proposed power adaptation scheme respectively improves the average network capacities by 0.2 Mbps, 0.1 Mbps and 0.05 Mbps (i.e. $\sim$%15 or more).

V. Conclusions

In this paper we first developed a link capacity model for CSMA based random networks that employ physical carrier sense and transmit power adaptation. Then using this model we constructed an optimization problem which maximizes the network capacity by finding the optimal power adaptation policy $\lambda^*(d)$ with the corresponding optimal physical carrier

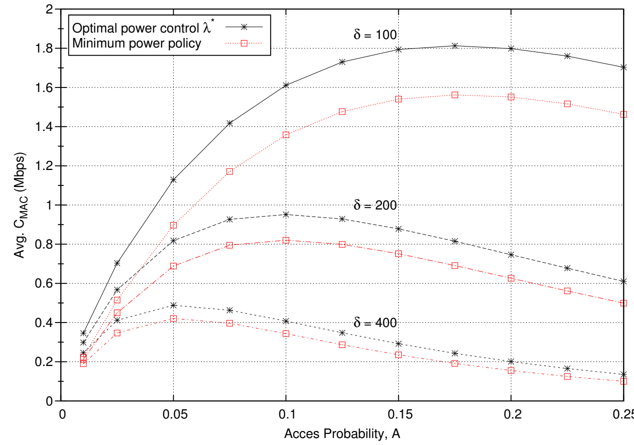

Fig. 7. Joint optimization of transmit power control policy $\lambda^*(d)$ and MAC layer A^* for $\delta = [100, 200, 400]$ links/km^2

sense threshold CS_{th}. Thereafter, using the proposed model, we showed that joint optimization of MAC parameter A and transmit power $\lambda(d)$ improves the network capacity. Finally comparing the proposed power adaptation scheme to the *minimum power policy* [1], we showed that it is possible to obtain more than $\sim$%15 increase in network capacity for networks with various densities δ.

The imminent next step on this research will be the verification of the theoretical results by means of simulations. A consequent research direction will be utilizing the proposed capacity model for the distributed and cross-layer optimization of wireless ad hoc networks.

References

[1] Y. Yang, J.C. Hou, and L.-C. Kung, "Modeling the effect of transmit power and physical carrier sense in multi-hop wireless networks," *Proc. IEEE INFOCOM*, May 2007.

[2] P. Gupta and and P. R. Kumar, "The capacity of wireless networks," *IEEE Transactions on Information Theory*,vol. 46, pp. 388–404, Mar. 2000.

[3] C. T. Lau and C. Leung, "Capture models for mobile packet radio networks," *IEEE Trans. on Communications*, vol. 40, no. 5, 1992.

[4] A. Papoulis, *Probability, random variables, and stochastic processes.* Mc-Graw Hill, 1984.

[5] F. Calì, M. Conti, and E. Gregori, "Dynamic Tuning of the IEEE 802.11 protocol to achieve a theoretical throughput limit," *IEEE/ACM Trans. Networking*, vol. 8, no. 6, pp. 785–799, Dec. 2000.

[6] Liang-liang Xie and P. R. Kumar, "A network information theory for wireless communication: Scaling laws and optimal operation," *IEEE Transactions on Information Theory*, vol. 4, pp. 748–767, May 2004.

[7] M. Gastpar and M. Vetterli, "On the capacity of wireless networks: The relay case," in *Proc. IEEE INFOCOM*, New York, NY, June 2002.

[8] F. Ye, S. Yi, and B. Sikdar, "Improving spatial reuse of IEEE 802.11 based ad hoc networks," in *Proc. IEEE GLOBECOM*, pp. 1013-1017, San Francisco, CA, Dec. 2003.

[9] N. Ehsan and R. Cruz, "On the optimal SINR in random access networks with spatial reuse," in *Proc. IEEE CISS*, Princeton, NJ, USA, Mar. 2006.

[10] J. Deng, B. Liang, and P. K. Varshney, "Tuning the carrier sensing range of IEEE 802.11 MAC," in *Proc. of IEEE GLOBECOM*, pp. 2987-2991, Dallas, TX, Dec. 2004.

[11] X. Yang and N. H. Vaidya, "On the physical carrier sense in wireless ad hoc networks," in *Proc. of IEEE INFOCOM*, Miami, Apr. 2005.

[12] T.-S. Kim et. al. "Improving spatial reuse through tuning transmit power, carrier sense threshold, and data rate in multihop wireless networks," in *Proc. of ACM MOBICOM*, Los Angeles, CA, Sep. 2006.